Phenomenological Loss Equivalence Method for Planar Quasi-TEM Transmission Lines with a Thin Normal Conductor or Superconductor

HAI-YOUNG LEE, STUDENT MEMBER, IEEE, AND TATSUO ITOH, FELLOW, IEEE

***Abstract* —The incremental inductance rule for conductor loss calculations is not valid if conductor thickness decreases and becomes comparable to the penetration depth, such as in monolithic microwave integrated circuits. In this paper, a simple approach, referred to as the phenomenological loss equivalence method, is proposed for characterizing a planar quasi-TEM transmission line with a thin normal conductor or superconductor over a wide range of field penetrations. For microstrip lines with a thin copper or high-T_c superconductor, the conductor losses calculated by this method agree very well with the published data calculated by the finite element method and the Monte Carlo method, respectively. Because of the simplicity of the calculation, this method should be very useful for the computer-aided design of monolithic microwave circuits.**

I. Introduction

HIGH SPEED and high degrees of integration in modern integrated circuits, especially in monolithic microwave integrated circuits, require very narrow and thin metal interconnection lines. High-speed electro-optic devices, such as traveling-wave optical modulators [1], use very narrow, thin electrodes for a very wide bandwidth. In a thin normal conductor line, the field penetration depth, which decreases with frequency due to the skin effect, can be comparable to the line thickness even at high frequencies, where the conductor loss becomes important. Therefore, the field penetration effect should be taken into account in the wide-band characterization of the lines. On the other hand, superconductors have very high conductivities and can be used to reduce the conductor losses of lines having a small cross section [2]. However, if the thickness or the width of the superconducting line is small enough to allow the field to penetrate deep into the line, the conductor loss will be increased and the merit of the nearly nondispersive transmission characteristic will be lost.

Therefore, for both the small normal-state and the superconducting line, the conductor loss must be characterized for a wide range of field penetrations by taking the field penetration effect into consideration. However, dc calculation at complete penetration and the incremental inductance method at shallow penetration [3] cannot be used directly over a wide range of field penetrations. The dc calculation assumes a uniform current distribution inside the conductor at the complete field penetration. On the other hand, the incremental inductance method is valid only if the conductor thickness is several times the penetration depth. Therefore, in order to consider the penetration effect, several loss analyses have been carried out for microstriplike structures. A variational formulation of the penetration-effect problem [4] was used to calculate the conductor loss of a single thin strip which has a rectangular cross section without a substrate and a ground plane. The finite element method was also applied to microstrip-like transmission line structures to calculate the resistance and reactance [5], [6]. However, these methods are not appropriate for computer-aided design implementation, since they require extensive formulations and numerical computations. A simple modification of the penetration-effect resistance [7] is valid only for a strip that is very thin and wide.

In this paper, we present a simple and practical method for the conductor loss calculation for a wide range of field penetrations which can be applied to normal-state and superconducting transmission lines. This phenomenological loss equivalence method (PEM) is based on the observation of the change in current distribution as the quasi-static field of a quasi-TEM transmission line penetrates into the conductor. In this method, a planar quasi-TEM transmission line, having a finite conductor thickness on the order of the penetration depth, is approximated by an equivalent single strip which has the same conductor loss as the transmission line. The geometry of the equivalent strip is obtained only from the cross-sectional geometry of the original line, while the conducting material is the same as that of the original line. Since the geometry of the equivalent strip obtained is independent of material properties, it can be used for any normal-state or superconducting line by substituting the conducting material. Therefore, the distributed internal impedance of the transmission line can be approximately calculated from the equivalent single

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The authors are with the Department of Electrical and Computer Engineering, University of Texas at Austin, Austin, TX 78712.
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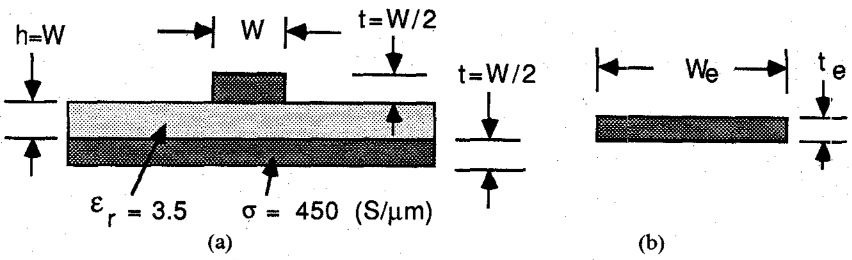


Fig. 1. (a) Copper microstrip line at 77 K and (b) the equivalent strip.

strip and all the propagation characteristics can be obtained by incorporating the internal impedance into the circuit model of the transmission line.

II. Phenomenological Loss Equivalence Method

A normal-state conductor has a skin depth (δ) which is inversely proportional to the square root of the frequency. Then, the field penetrates deeper at lower frequencies and the current flowing inside the conductor becomes more uniform. On the other hand, a superconductor has an almost constant penetration depth (λ) at a given temperature below the critical temperature (T_c) and the field penetration remains almost unchanged with the frequency. Based on the two-fluid model [8], a superconductor can be described as a conductor which has the complex conductivity ($\sigma = \sigma_n + j\sigma_s$, where σ_n and σ_s are the normal conductivity and superconductivity, respectively) depending on the temperature and the frequency. Therefore, by assuming a generalized complex conductivity ($\sigma = \sigma_r + j\sigma_i$), we can treat both normal conductors and superconductors with an identical formulation.

For a given planar quasi-TEM transmission line such as the microstrip line shown in Fig. 1(a), if the penetration depth is very shallow compared to the conductor thickness, the surface current distribution will be the same as that of the ideal microstrip, and we can then apply the incremental inductance rule to the structure. On the other hand, if the current completely penetrates into the conductor, the current distribution will be uniform inside the conductor. Now, if the penetration is moderate, the current distribution becomes nonuniform on the surface and decays almost exponentially from the surface into the conductor. The field penetrations result in the distributed resistance (R) and internal inductance (L_i) which will be added in series to the external inductance (L) of the ideal line. The distributed internal impedance ($Z_i = R + j\omega L_i$) causes the conductor loss and the dispersion of the transmission line, which degrade the transmission characteristics.

For the nonuniform current distribution of the transmission line due to the penetration, we introduce the equivalent single strip shown in Fig. 1(b), which is assumed to have the same internal impedance as the original line. The surface current on the equivalent strip is assumed to be uniform horizontally, but not vertically. The equivalent strip has the same conducting material as the original line while the width and thickness of the equivalent strip will be obtained independently of the conducting material used. In order to obtain the equivalent strip, we consider the two completely different cases of shallow penetration and complete penetration.

First, the equivalent width (W_e) can be calculated, at a very shallow penetration, by equating the distributed internal impedance of the original line to that of the equivalent strip. It is well known that the internal impedance of a transmission line can be calculated using the incremental inductance rule [3] if the penetration depth (δ, λ) into the conductor is very shallow compared to the conductor thickness (t), or $\delta, \lambda/t \ll 1$. Specifically, it can be expressed in the simple form

$$Z_i = Z_s G \qquad (\Omega/\text{m}) \tag{1}$$

where $Z_s (= \sqrt{j\omega\mu/\sigma})$ is the surface impedance (Ω/square) of the conductor used and G is a geometric factor with dimension m^{-1}. For instance, the Z_s is $(1+j)/\sigma\delta$ or $\sim 1/\sigma\lambda$ for a normal conductor or a superconductor, respectively. From the incremental inductance rule, the G can be expressed in terms of the incremental inductance associated with the penetration of magnetic flux into the conductor. That is,

$$G = \frac{1}{\mu} \sum_j \frac{\partial L}{\partial n} \tag{2}$$

where $\partial L/\partial n$ is the derivative of the external inductance L with respect to the incremental recession of the conductor wall j. Here, the mean depth of the current penetration, used in the incremental inductance rule, is half of the skin depth for a normal conductor and the penetration depth for a superconductor. As an example, $G = (2/\text{strip width})$ for a very wise microstrip line [6]. The factor G depends only on the cross-sectional geometry of the transmission line, that is, the quasi-static surface current distribution for the case of an ideal line.

Under this condition of shallow field penetration, we also apply the incremental inductance rule to the equivalent strip. Since the current distribution is assumed to be uniform on the equivalent strip, the distributed internal impedance is just the surface impedance divided by the equivalent strip width. Then,

$$Z_i = Z_s / W_e \qquad (\Omega/\text{m}) \tag{3}$$

where W_e is the equivalent width and Z_s is the surface impedance of the conductor used in the original line. Therefore, by equating (1) and (3), we can obtain the equivalent width at the shallow penetration as

$$W_e = 1/G \qquad (\text{m}). \tag{4}$$

The W_e obtained from the incremental inductance rule is dependent only on the structure and does not depend on the conducting material used.

Now, if the field penetrates deep into the conductor, the distributed internal impedance will also depend on the strip thickness. However, if the current distribution on the surface of the transmission line can be assumed to be almost unchanged, e.g., in the case of quasi-TEM transmission line, we can still use the equivalent width (W_e) calculated above and include the field penetration effect in the conductor thickness (t_e) of the equivalent strip. The equivalent thickness (t_e) will be obtained by equating both internal impedances in the case of uniform current distribution at complete penetration. For a uniform distribution of current, the distributed internal impedance of the original line can be simply expressed as

$$Z_i = 1/(\sigma A) \tag{5}$$

where A is the effective cross section of the original structure at the uniform current distribution. For instance, A is just the cross section (Wt) of the microstrip conductor because of the infinite ground plane. At the same complete penetration, the equivalent strip has an internal impedance as follows:

$$Z_i = 1/(\sigma W_e t_e). \tag{6}$$

Then, using (5) and (6), we can find the equivalent thickness (t_e) as

$$t_e = A/W_e = AG. \tag{7}$$

Here W_e and t_e depend only on the cross-sectional geometry of the original transmission line, since G and A can be calculated from the cross-sectional geometry. Therefore, an equivalent strip obtained from a given geometry of the transmission line can be used approximately for any conducting material and penetration depth.

Now, using the equivalent strip obtained from the cross-sectional geometry and the conductivity of a given transmission line, we can calculate the overall distributed internal impedance at any field penetration. For the equivalent strip with a laterally uniform current distribution, the overall distributed internal impedance can be calculated using the equivalent strip width (W_e) and the surface impedance (Z_s^t) of a flat plane conductor with finite thickness [9]. In the surface impedance calculation, the longitudinal current distribution in the vertical direction of the strip is subject to the boundary conditions at the two strip surfaces. From these conditions, the longitudinal current (I) integrated through the strip thickness can be obtained in terms of the longitudinal electric field (E_0) on the strip surface. Therefore, the surface impedance (Z_s^t) can be expressed with a correction factor for finite strip thickness as follows:

$$Z_s^t = E_0/I = Z_s \coth[\tau t_e] \quad (\Omega/\text{square}) \tag{8}$$

where $\tau(=[j\omega\mu\sigma]^{1/2})$ is a complex decaying constant associated with the vertical variation of the field inside the strip. For a strip using a normal conductor, the conductivity (σ) is pure real; the surface impedance (Z_s^t) can then be written in terms of the surface resistivity ($R_s = 1/\sigma\delta$) and the skin depth (δ) as follows:

$$Z_s^t = (1+j)R_s \coth[(1+j)t_e/\delta] \quad (\Omega/\text{square}). \tag{9}$$

The surface impedance in (8) incorporates the field penetration effect using the correction factor ($\coth[\tau t_e]$). Since the laterally uniform current of the equivalent strip extends over the finite width $W_e(=1/G)$, the distributed internal impedance (Z_i) of the equivalent strip becomes $Z_s^t/W_e(=Z_s^t G)$ by replacing the surface impedance in (3) with (8).

Finally, for a planar quasi-TEM transmission line with finite conductor thickness, the distributed internal impedance at any field penetration can be approximately calculated through the equivalent strip as

$$Z_i = R + j\omega L_i = Z_s^t/W_e = Z_s G \coth[\tau GA] \quad (\Omega/\text{m}). \tag{10}$$

This can be rewritten for a planar quasi-TEM transmission line using a normal conductor as follows:

$$Z_i = (1+j)R_s G \coth[(1+j)GA/\delta] \quad (\Omega/\text{m}). \tag{11}$$

The propagation characteristics (i.e., attenuation, effective index, and characteristic impedance) of the original transmission line can be readily calculated from a circuit model of the transmission line. In the model, the distributed resistance (R) and the internal inductance (L_i) calculated from (10) will be added in series to the external inductance (L) of the transmission line, while the shunt capacitance (C) remains almost constant for the field penetration. Then, we can use general formulas for the circuit model to calculate the propagation characteristics, where the conductance for dielectric loss can also be incorporated through the calculation of effective loss tangent [7], [10].

The usefulness of this method comes from the very simple calculations of G and A used in (10). Here, G and A of a transmission line can be calculated using the empirical formulas and from the actual cross section of the line, respectively. Also, Z_s and τ can be easily obtained from the conductivity of the conducting material used. Therefore, all the calculations consist only of simple evaluations of several formulas. This method can also be applied to any quasi-TEM transmission line, i.e., microstrip, coplanar waveguide, coplanar strip, and so forth.

III. Calculated Results for Microstrip Lines

In order to verify the proposed phenomenological loss equivalence method, we applied the method to microstrip lines using copper conductor and high-T_c superconductor and compared the calculated data with published data.

A. Calculated Results for a Copper Microstrip

For the copper microstrip line shown in Fig. 1, many transmission characteristics were calculated using the PEM over a wide frequency range and for a wide range of geometrical dimensions. In Fig. 2, the results are compared

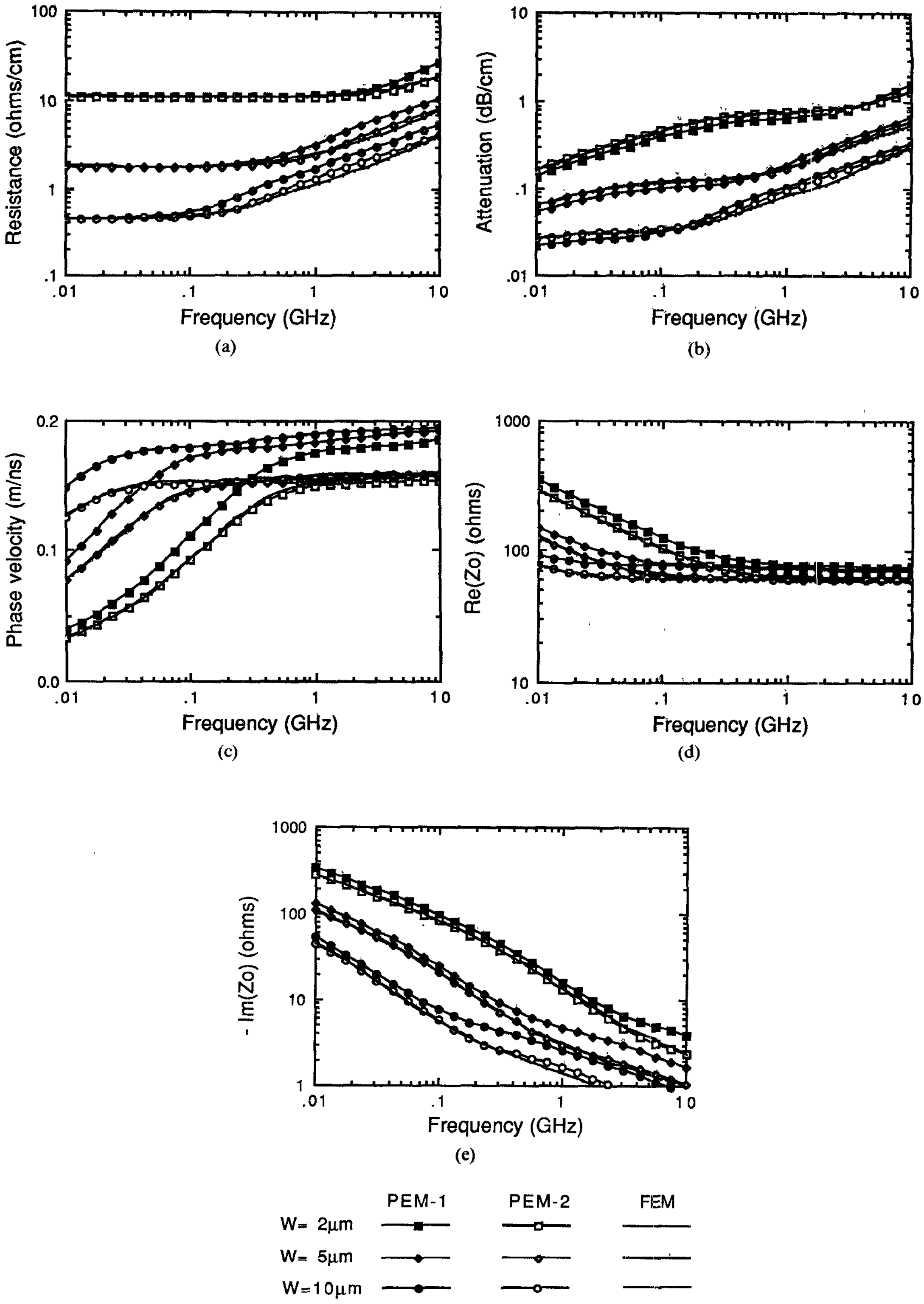


Fig. 2. (a) The distributed resistance, (b) the attenuation, (c) the phase velocity, and (d) the real and (e) imaginary parts of the characteristic impedance of the copper microstrip line shown in Fig. 1 for different strip widths at 77 K. PEM-1 and PEM-2 show the PEM data calculated based on the empirical formulas and on the geometric factors and effective indices obtained from the FEM data at high frequency, respectively.

with the published data calculated by the finite element method (FEM) [6]. One set of PEM data (PEM-1 in Fig. 2) are calculated by directly applying the method to the microstrip on the basis of the available empirical formulas for the microstrip [11]. Note that the microstrip used in the FEM analysis [6] has the strip conductor embedded in the infinitely thick dielectric poliymide, while in this PEM calculation we consider a real situation wherein the strip conductor lies just on top of the finite dielectric. Therefore, the phase velocities of the FEM data at high frequency approach the velocity of light in the dielectric medium and are higher than those calculated from the empirical formulas. In addition, since the empirical formulas used in the PEM calculation are not accurate at this high ratio of thickness to width of the microstrip conductor, the geometric factors (G) calculated using the incremental inductance rule are inaccurate. Therefore, more accurate G's are calculated from the FEM data at the highest frequency (10 GHz) because the field penetration at the frequency will be very shallow and the distributed internal impedance (Z_i) can be expressed simply as Z_sG.

In order to compare the PEM data with the FEM data in the same situation and verify the PEM itself, another set of PEM data (PEM-2 in Fig. 2) were generated, based on the more accurate geometric factors and the effective indices mentioned above. As shown in Fig. 2, all the PEM data of the second set are in excellent agreement with the FEM data for wide ranges of frequency and geometrical dimension. This indicates the validity of the PEM for the conductor loss calculation for a wide range of field penetrations. There are two deflection points for each attenuation curve shown in Fig. 2(b). The deflection at the upper frequency is associated with the saturation of the field penetration due to the finite strip thickness. Another deflection comes out at the lower frequency where the total reactance ($j\omega[L+L_i]$) and the resistance (R) become about the same. A wide and flat attenuation region exists between the two frequencies and can be shifted to the lower frequency by increasing the conductivity or the cross section of the line. These deflection points will exist in all planar quasi-TEM transmission lines with normal conductors.

The distributed resistance (R) of the first PEM set (PEM-1) shows a slight difference at high frequency due to the inaccurate calculation of G values. The inaccurate calculation of G can easily be rectified if we use more accurate empirical formulas for the microstrip. In spite of the slightly inaccurate G's, all of the PEM-1 data agree well with the FEM data except for the effective indices due to the different dielectric thicknesses considered.

B. Calculated Results for a Superconducting Microstrip Line

A thin microstrip line having the geometry of Fig. 3 is chosen for an application of the PEM to superconducting line. In [12], the Monte Carlo method (M.C.) is used to analyze the microstrip line with aluminum or a high-T_c superconductor (Ba–Y–Cu–O). The high-T_c superconductor is described using the two-fluid model in both this PEM and the M.C., although the validity of this model in high-T_c material is uncertain. It is used here only to make a comparison of the PEM data with the M.C. data. Using the PEM, the attenuations and the phase velocities are calculated and compared with the data from the M.C. method in Fig. 4. They are in very good agreement over a wide range of frequencies and show the validity of the PEM for a superconducting line. The superconducting line shows very small attenuation and virtually nondispersive transmission, while the aluminum line is very lossy at high frequencies and dispersive in the low-frequency region. Although the superconducting line has an attenuation that is proportional to the square of the frequency, it is almost ideal for transmission except at very high frequencies close to the band gap frequency.

Since the penetration depth of the superconductor at 77 K (0.2 μm) is not so deep compared to the microstrip thickness (0.5 μm), the penetration effect cannot be seen clearly in this geometry. The penetration effect rather can be seen clearly in the aluminum line as well as in the previous copper line, because their skin depths are frequency-dependent and hence are very great at low frequency. Therefore, the PEM will be more effective for very thin and narrow superconducting lines. For those small lines, the line thicknesses are not thick enough to apply the

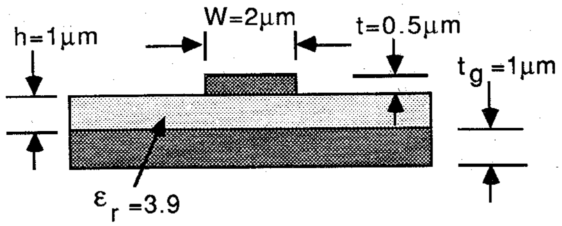
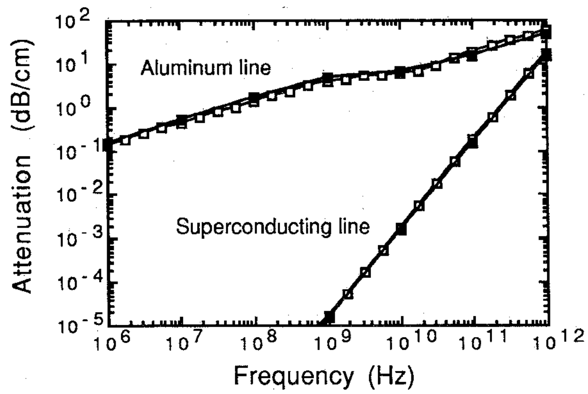
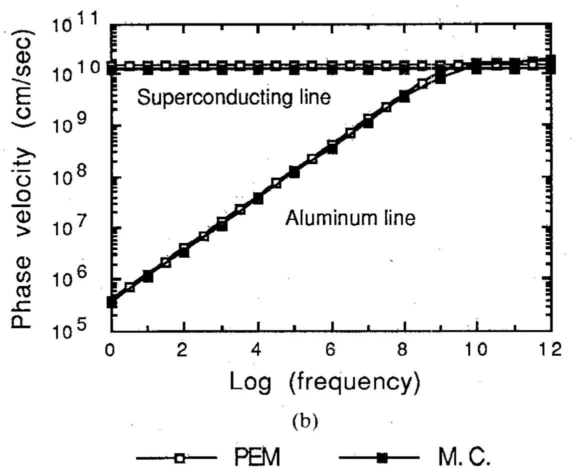


Fig. 3. Microstrip line using a high-T_c superconductor ($T_c = 92.5$ K, penetration depth at 0 K $= 0.14$ μm, and normal conductivity at $T_c = 0.5$ S/μm) or aluminum (conductivity at 77 K $= 150$ S/μm).



(a)



(b)

Fig. 4. (a) The attenuation and (b) the phase velocity of the superconducting and aluminum microstrip lines at 77 K shown in Fig. 3.

incremental inductance method and a simple modification of the surface impedance [2], [13], [14] will be inaccurate due to the lateral surface current spreading at very deep field penetrations.

IV. Conclusion

A phenomenological loss equivalence method is proposed for characterizing the conductor loss of a planar quasi-TEM transmission line made of a normal conductor or superconductor over a wide range of field penetrations. In this method, the distributed internal impedance of the line is readily calculated from a single strip equivalent to the original transmission line. In order to verify the method, we applied it to thin microstrip lines made of thin copper and high-T_c superconductor. The calculated results show very good agreement with those calculated using the finite element method and the Monte Carlo method over wide ranges of field penetration and geometrical dimension. Due to the simplicity of the calculation, this method should be very useful in the computer-aided design of monolithic microwave circuits.

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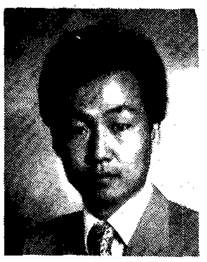

Hai-Young Lee (S'89) was born in Seoul, Korea, on March 7, 1957. He received the B.S. degree in electronic engineering from Ajou University, Korea, in 1980 and the M.S. degree in electrical engineering from the Korea Advanced Institute of Science and Technology, Korea, in 1982.

From 1982 to 1986, he was a senior research engineer in the Ministry of National Defense, Korea, where he worked on the numerical analysis of wide-band antennas and electromagnetic compatibility. Since 1986, he has been with the Electrical and Computer Engineering Department, University of Texas at Austin, where he is currently pursuing the Ph.D. degree. His current interests are in high-speed optoelectronic devices, integrated microwave circuits, and superconductor applications for high-density interconnection lines.

Tatsuo Itoh (S'69–M'69–SM'74–F'82) received the Ph.D. degree in electrical engineering from the University of Illinois, Urbana, in 1969.

From September 1966 to April 1976, he was with the Electrical Engineering Department, University of Illinois. From April 1976 to August 1977, he was a Senior Research Engineer in the Radio Physics Laboratory, SRI International, Menlo Park, CA. From August 1977 to June 1978, he was an Associate Professor at the University of Kentucky, Lexington. In July 1978, he joined the faculty at the University of Texas at Austin, where he is now a Professor of Electrical Engineering and Director of the Electrical Engineering Research Laboratory. During the summer of 1979, he was a guest researcher at AEG-Telefunken, Ulm, West Germany. Since September 1983, he has held the Hayden Head Centennial Professorship of Engineering at the University of Texas. In September 1984, he was appointed Associate Chairman for Research and Planning of the Electrical and Computer Engineering Department. He also holds an Honorary Visiting Professorship at the Nanjing Institute of Technology, China.

Dr. Itoh is a member of Sigma Xi and of the Institute of Electronics and Communication Engineers of Japan. He is a member of Commission B and Chairman of Commission D of USNC/URSI. He served as the Editor of the IEEE Transactions on Microwave Theory and Techniques during the years 1983–1985. He serves on the Administrative Committee of the IEEE Microwave Theory and Techniques Society. Dr. Itoh is a Professional Engineer registered in the state of Texas.